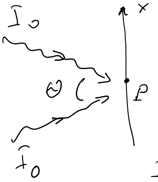
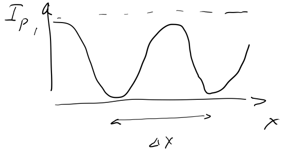
Interference of quasi monochromatic light. Time coherence.

What does it mean quasi? Quasi means here „almost". So far we have been discussing monochromatic light



$I_1 = I_2 = I_0$

$I_p = 2I_0(1+\cos k_0\Delta)$

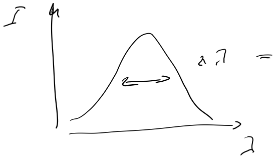


$V=1$

$\Delta x = \frac{\lambda}{\theta}$

This model that we built for monochromatic light is far away from reality. The real source does not emit monochromatic light.

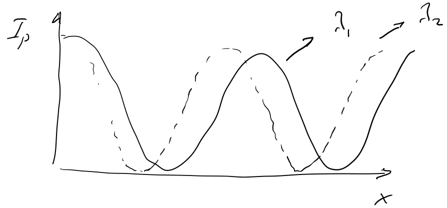
Atoms emit some spectrum:

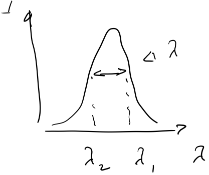


$\Delta\lambda \Rightarrow$ variations in $\Delta x \Rightarrow V\downarrow$

Qualitative approach

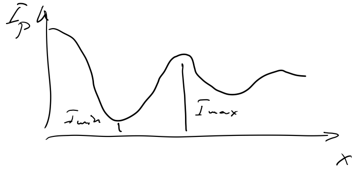
① Since now we have some distribution $\Delta\lambda$, Δx will be also distributed:





On a screen we will have overlap of two interference patterns. This will be simple sum of intensities as the waves are not coherent.

The visibility of such interference will be smaller:

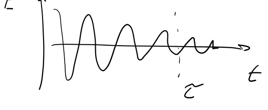


$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

② We remember that time of atom emission is finite and about 10^{-8} s for natural broadening.

$$\boxed{\Delta\nu \cdot \tau \approx 1}$$

$$\nu = \frac{c}{\lambda} \qquad |\Delta\nu| = \frac{c\,\Delta\lambda}{\lambda^2}$$

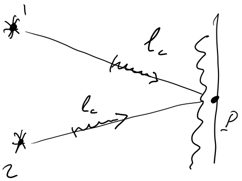


$$\frac{\Delta\lambda}{\lambda^2}\cdot\underbrace{c\tau}_{l_c \sim \text{light train length}}=1 \qquad l_c=\frac{\lambda^2}{\Delta\lambda}$$

$l_c = c\cdot\tau$

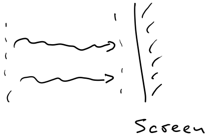
So if atom emission life time is 1 ns $l_c \sim 30$ cm.

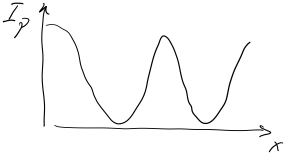
Please, also remember, each train has its own initial phase.



Here we can have the following situations:

1)





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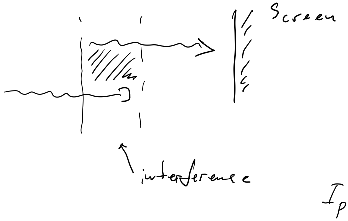
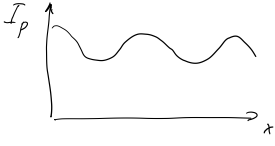
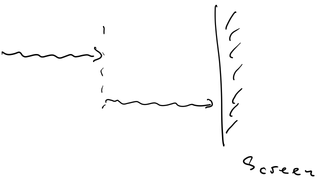
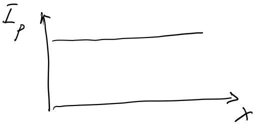




3)

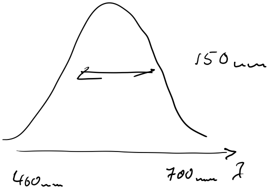




Time coherence is the ability of two light trains, that have certain optical path difference, create interference pattern.

If l_c is big (like HeNe), then time coherence is large, if l_c is short (like fs pulses) then time coherence is small.

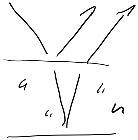
For Sun light:



$$l_c = \frac{\lambda^2}{\Delta\lambda} \approx 2\mu m$$

In fact Sun l_c is <10fs.

So, if we take thin film:



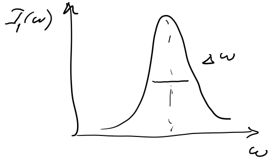
$$\Delta_{max} \approx 2dn$$

$$d_{max} = \frac{\Delta_{max}}{2n}$$ (0.1 in sunlight)

Quantitative approach

How V is connected with spectral composition of radiation?

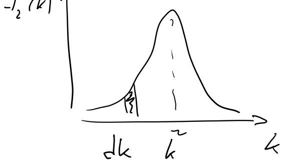
Let's take non-monochromatic radiation!



$\Delta\omega$ is not big as we consider quasi monochromatic light.

$$I_1(\omega) \Rightarrow I_2(k)$$

$$k = \frac{\omega}{c}$$



Let's select monochromatic component with very narrow variation of k.

$$dI = I_2(k)dk$$

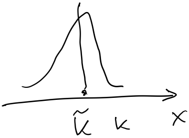
$$dI_p = 2I_2(k)\,(1+\cos k\Delta)\,dk$$

This is interference pattern from single dk area. This pattern is strong and bright with visibility $V=1$.

Now, we sum such interference patterns from other dk areas. Since they are not coherent, this is simple summation of interference patterns .

$$I_p(\Delta) = 2\int I_2(k)\,(1+\cos k\Delta)\,dk$$

$I_2(k) = f(x)$ $x = k-\tilde{k}$



$$I_p(\Delta) = 2\int_{-\infty}^{+\infty} f(x)\left[1+\cos[(x+\tilde{k})\Delta]\right]dx =$$

$$= 2\int_{-\infty}^{+\infty} f(x)\,dx + 2\int_{-\infty}^{+\infty} \underbrace{f(x)\cos x\Delta\,dx \cdot \cos\tilde{k}\Delta}_{A} -$$

$$-2\int_{-\infty}^{+\infty} \underbrace{f(x)\sin x\Delta\,dx \cdot \sin\tilde{k}\Delta}_{\overset{\shortparallel}{0}}$$

$(\sin x$ is odd function$)$

We have two terms left. Let's work with them.

$$I_0 = \int_{-\infty}^{+\infty} f(x)\, dx$$

$$I_p = 2I_0 + 2 \int_{-\infty}^{+\infty} f(x) \cos x\Delta\, dx \cdot \cos \tilde{k}\Delta =$$

$$= 2I_0 \Bigg(1 + \underbrace{\frac{\int_{-\infty}^{+\infty} f(x) \cos x\Delta\, dx}{I_0}}_{C(\Delta)} \cos \tilde{k}\Delta \Bigg) =$$

$$= 2I_0 \left(1 + C(\Delta) \cos \tilde{k}\Delta \right)$$

$$C(\Delta) = \frac{\int_{-\infty}^{+\infty} f(x) \cos x\Delta\, dx}{I_0}$$

Please, note, for monochromatic light $C(\Delta) = 1$ and visibility $V = 1$.

What would be visibility now?

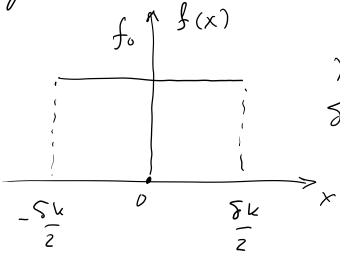
Let's calculate I_{max} and I_{min}.

$$\left.\begin{aligned} I_{max} &= 2I_0 \left(1 + |C(\Delta)| \right) \\ I_{min} &= 2I_0 \left(1 - |C(\Delta)| \right) \end{aligned}\right\} \quad V = |C(\Delta)|$$

What would be a conclusion here?

V depends on $f(x)$, hence depends on $I_1(\omega)$.

Let's have a look at an example. Imagine, we observe interference from radiation with rectangular spectral width:



$x = k - \tilde{k}$

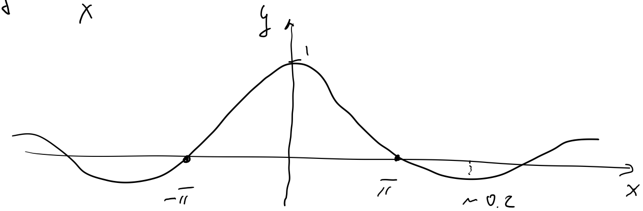
$\delta x = \delta k.$

$V => C(\Delta)$

$$C(\Delta) = \frac{\int_{-\frac{\delta k}{2}}^{\frac{\delta k}{2}} f_0 \cos x\Delta \, dx}{\int_{-\frac{\delta k}{2}}^{\frac{\delta k}{2}} f_0 \, dx} =$$

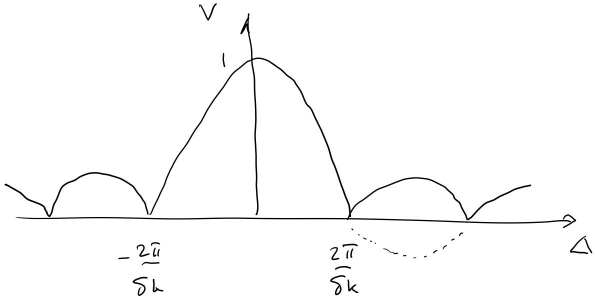
$$= \frac{\sin\left(\frac{\delta k \cdot \Delta}{2}\right)}{\frac{\delta k \cdot \Delta}{2}} = \operatorname{sinc}\left(\frac{\delta k \cdot \Delta}{2}\right)$$

$$y = \frac{\sin X}{X}$$



Now, let's find modulus of this function.

$$V = |c(\Delta)|$$



$$\Delta_{max} = \frac{2\pi}{\delta k} \qquad k = \frac{2\pi}{\lambda} \qquad |\delta k| = \frac{2\pi}{\lambda^2}\delta\lambda$$

$$\Delta_{max} = \frac{\lambda^2}{\delta\lambda} - l_c - \text{coherence length}$$

Now we see, that if Δ is greater than l_c then we will see simple averaged intensity.

Demonstration of thin film

Why film was white at the beginning?